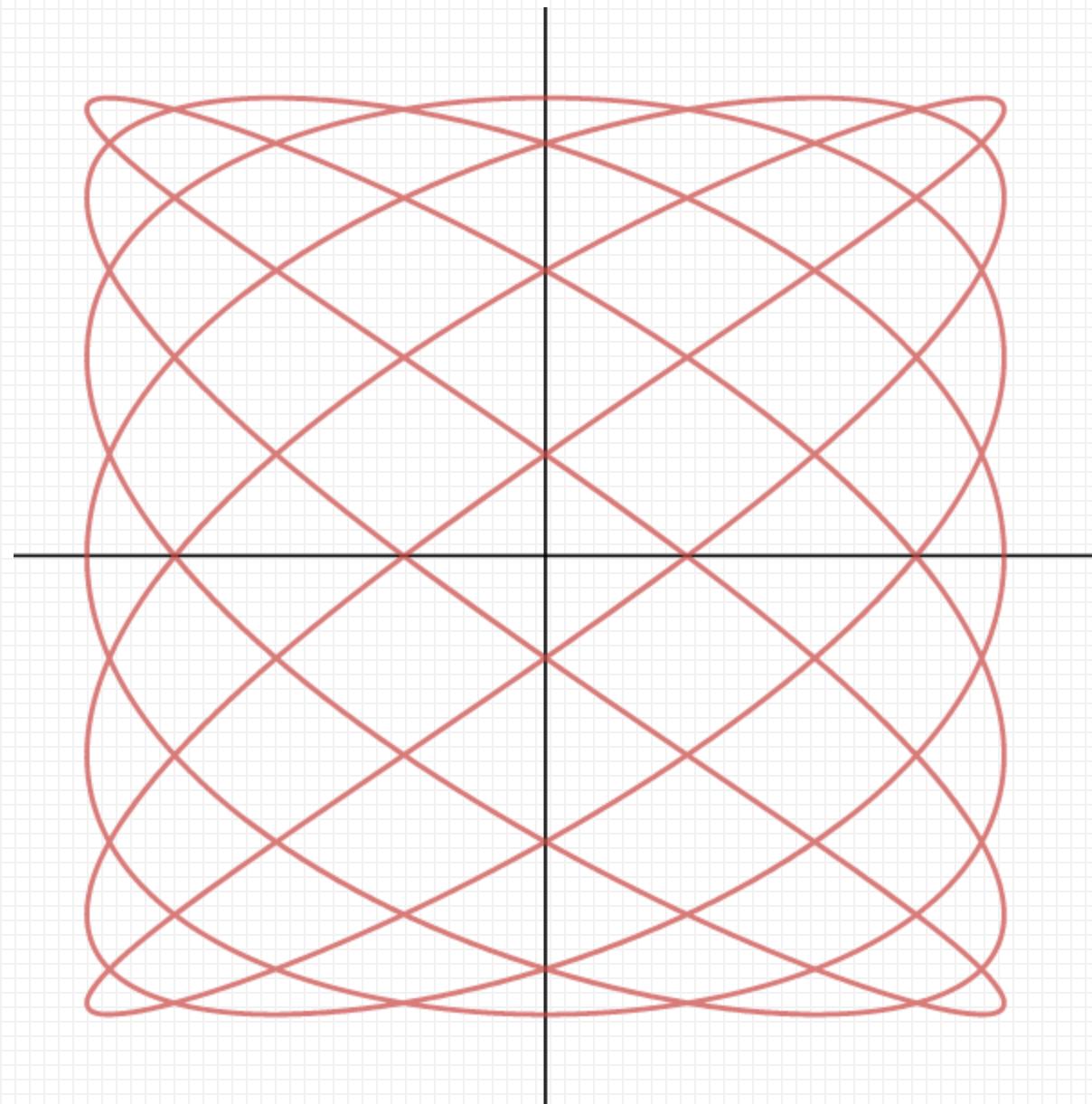


# **Section 10.1**

## **Arc Length**



**The equation for the length of a parametric curve is like our previous “length of curve” equation:**

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

(Hmm.....smells like the Pythagorean Theorem.)



## Derivation:

Fix  
Mode

$$\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\int \sqrt{1 + \frac{\left(\frac{dy}{dt}\right)^2}{\left(\frac{dx}{dt}\right)^2}} dx$$

$$\int \frac{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}{\left(\frac{dx}{dt}\right)^2} dx$$

$$\int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot \frac{dx}{dt}$$

$$\boxed{\int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt}$$



$(\sin t)^3$

Find the length of the curve  $x = \cos^3 t$ ,  $y = \sin^3 t$ , on  $[0, 2\pi]$ .

$$\frac{dx}{dt} = -3\cos^2 t \sin t, \frac{dy}{dt} = 3\sin^2 t \cos t$$

$$L = \int_0^{2\pi} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t} dt$$

$9 \cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)$

$$L = \int_0^{2\pi} |3\cos t \sin t| \sqrt{\sin^2 t + \cos^2 t} dt$$

$$L = \int_0^{2\pi} |3\cos t \sin t| dt$$



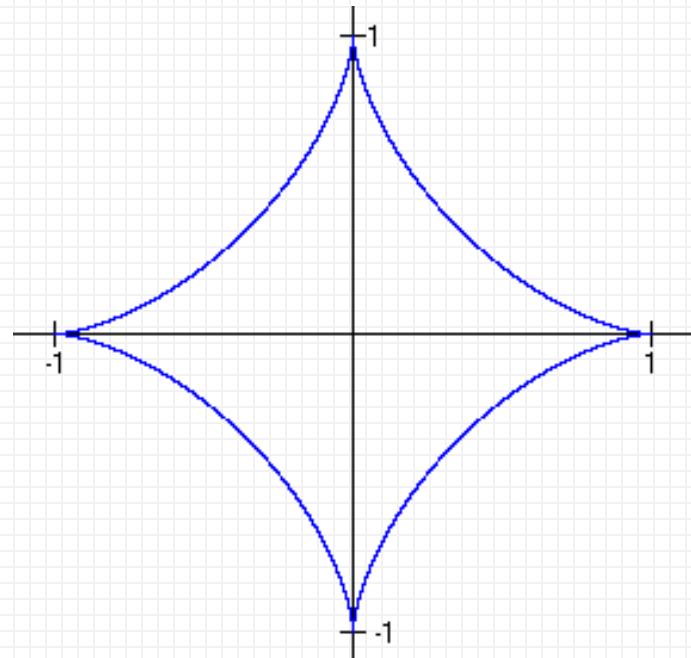
$$L = \int_0^{2\pi} |3\cos t \sin t| dt$$

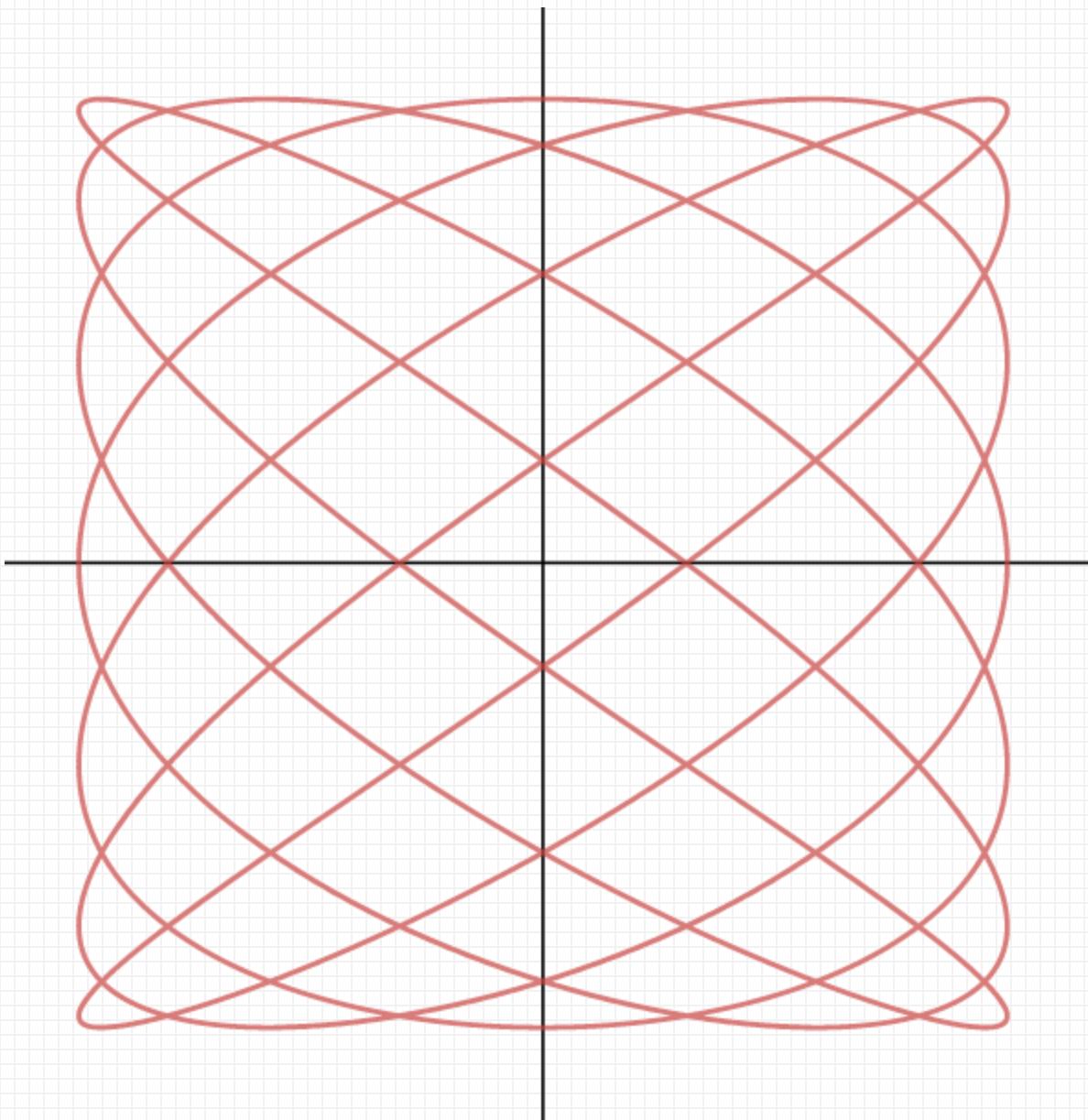
$$L = 4 \int_0^{\pi/2} 3\cos t \sin t dt$$

$$u = \sin t$$

$$du = \cos t dt$$

$$L = 12 \int_0^1 u du = 6u^2 \Big|_0^1 = 6$$





This curve is:

$$x(t) = \cos(7\pi t)$$

$$y(t) = \sin(5\pi t)$$

# **Homework:**

**p. 518 # 11 – 15 odd; AP Packet #18-22**

